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Hence, the required equation is

$$x^3 - \tfrac{3}{4}x - \tfrac{1}{8} = 0.$$

Solved similarly by H. C. Feemster, and also by C. E. Githens by merely substituting the values of a_1, a_2, a_3 and reducing by means of trigonometric relations. Mr. Feemster also sent in such a second solution.

Editorial note. The details of the work on this problem may be simplified by use of Gauss' periods. See Weber's *Algebra*, volume I, chapter XVI.

413. Proposed by C. N. SCHMALL, New York City.

Apply Euler's transformation to show that

$$1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots = \frac{1+x}{(1-x)^3}.$$

Solution by W. C. Brenke, University of Nebraska.

This is a special case of the series

$$S(x) = 1 + 2^rx + 3^rx^2 + 4^rx^3 + \cdots; \; r = \text{positive integer}.$$

Such series may be summed by repeated use of Euler's transformation based on the factor $(1 - x)$. The whole operation may be performed at once as follows, the process being valid for $| x | < 1$.

Multiply both sides of the last equation by

$$(1 - x)^{r+1} = 1 - {}_{r+1}C_1x + {}_{r+2}C_2x^2 - \cdots + (-1)^{r+1}x^{r+1}.$$

Arranging the right member in powers of x, we find that the coefficients of x^{r+1}, x^{r+2}, $\cdots$ are all zero by virtue of the formula (special case of (3) p. 183 of Chrystal's Algebra, Vol. 2)

$$n^r - (n-1)^r{}_nC_1 + (n-2)^r{}_nC_2 - \cdots + (-1)^{n-1}{}_nC_{n-1} = 0; \; r < n.$$

Hence

$$\begin{aligned}(1 - x)^{r+1}S(x) = 1 + (2^r - {}_{r+1}C_1)x + (3^r - 2^r{}_{r+1}C_1 + {}_{r+1}C_2)x^2 + \cdots \\ + (r^r - (r-1)^r{}_{r+1}C_1 + \cdots + (-1)^r{}_{r+1}C_r)x^r.\end{aligned}$$

The result may also be written in the form

$$S(x) = \frac{S_r(x) - {}_{r+1}C_1xS_{r-1}(x) + {}_{r+1}C_2x^2S_{r-2}(x) + \cdots + (-1)^r{}_{r+1}C_rx^r}{(1-x)^{r+1}},$$

where

$$S_n(x) = 1 + 2^rx + 3^rx^2 + \cdots + (n+1)^rx^n.$$

When $r = 2$ this gives the sum of the proposed series.

Also solved by the Proposer.

414. Proposed by R. D. CARMICHAEL, Indiana University.

Prove by means of an example that one of the series

$$\sum_{k=1}^{\infty} \frac{1}{c_k}, \quad \sum_{k=1}^{\infty} \frac{1}{c_k - 1}, \qquad c_k \neq 0, 1,$$

may be divergent while the other is convergent.

SOLUTION BY E. B. WILSON, Massachusetts Institute of Technology.

The following two series fit the conditions of the problem:

$$\frac{1}{\sqrt{2}}+\frac{1}{-\sqrt{3}}+\frac{1}{\sqrt{4}}+\frac{1}{-\sqrt{5}}+\cdots,$$

$$\frac{1}{\sqrt{2}-1}+\frac{1}{-\sqrt{3}-1}+\frac{1}{\sqrt{4}-1}+\frac{1}{-\sqrt{5}-1}+\cdots.$$

The first converges because it is an alternating series whose terms steadily decrease in absolute value and approach zero. That the second is divergent may be seen by bracketing the terms in pairs. For, if n is even, we have

$$\frac{1}{\sqrt{n}-1}+\frac{1}{-\sqrt{n+1}-1}=\frac{-\sqrt{n}+\sqrt{n+1}+2}{(\sqrt{n}-1)(\sqrt{n+1}-1)}>\frac{2}{n+1};$$

whence the series is divergent as one sees by comparison with the divergent series

$$\frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\cdots.$$

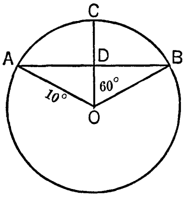
GEOMETRY.

440. Proposed by W. L. WATSON, Moundsville, West Va.

A solid sector is cut out of a sphere 10 feet in radius by a cone whose vertical angle is 120°. Find the radius of the sphere whose volume is equal to that of the sector.

SOLUTION BY EMMA M. GIBSON, Drury College.

The volume of the solid sector O–ABC is equal to that of the cone AOB, having a vertex angle of 120° and altitude OD equal to 10 cos 60°, plus the volume of a spherical segment of altitude $CD = 10 - 10\cos 60° = 5$, and diameter of base $AB = 2 \times 10\sin 60° = 10\sqrt{3}$. Then



$$\text{Volume of cone} = \frac{1}{3}\text{base} \times \text{altitude} = \frac{1}{3}\pi 75 \times 5 = 125\pi.$$

$$\text{Volume of segment} = \frac{1}{3}\pi \times 25 \times 25 = \frac{625}{3}\pi.$$

$$\therefore \text{Volume of solid sector} = 125\pi + \frac{625\pi}{3} = \frac{1000\pi}{3}.$$